

# Time variation of the electron mass in the early universe and the Barrow-Magueijo model

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## ABSTRACT

We put limits on the time variation of the electron mass in the early universe using observational primordial abundances of D,  $^4\text{He}$  and  $^7\text{Li}$ , recent data from the Cosmic Microwave Background and the 2dFGRS power spectrum. Furthermore, we use these constraints together with other astronomical and geophysical bounds from the late universe to test Barrow-Magueijo's model for the variation in  $m_e$ . From our analysis we obtain  $-0.615 < G\omega/c^4 < -0.045$  ( $3\sigma$  interval) in disagreement with the result obtained in the original paper.

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## 1. Introduction

Time variation of fundamental constants over cosmological time scales is a prediction of theories that attempt to unify all fundamental interactions like string derived field theories (Wu and Wang 1986; Maeda 1988; Barr and Mohapatra 1988; Damour and Polyakov 1994; Damour et al. 2002a,b), related brane-world theories (Youm 2001a,b; Palma et al. 2003; Brax et al. 2003), and Kaluza-Klein theories (Kaluza 1921; Klein 1926; Weinberg 1983; Gleiser and Taylor 1985; Overduin and Wesson 1997). In order to study the possible variation in the fine structure constant or the electron mass, theoretical frameworks based on first principles, were developed by different authors (Bekenstein 1982; Bekenstein 2002; Barrow et al. 2002; Barrow and Magueijo 2005).

The predicted time behaviour of the fundamental constants depends on which version of the theories is considered. Thus, bounds obtained from astronomical and geophysical data are an important tool to test the validity of these theories. In a previous work (Mosquera et al. 2007), we have analyzed the variation in the fine structure constant in the context of Bekenstein model. In this paper, instead, we study the variation in the electron mass ( $m_e$ ) in the context of the Barrow-Magueijo model (Barrow and Magueijo 2005). Note that  $m_e$  is not a fundamental constant in the same sense as the fine structure constant is. Hence, it could be argued that constraints on the time variation of the Higgs vacuum expectation value ( $\langle v \rangle$ ), rather than  $m_e$ , are more relevant. Moreover, the possibility of a time variation of the vacuum expectation value of a field seems more plausible than the time variation of a gauge coupling constant. However, in the context of the Barrow-Magueijo model, the relevant fundamental constant is  $m_e$  and thus we will focus on its possible variation. The joint variation in the fine structure constant and  $\langle v \rangle$  in the early universe will be analyzed in a forthcoming paper.

Constraints on  $m_e$  variation over cosmological time scales are available from astronomical and local methods. The latter ones include geophysical methods (analysis of natural long-lived  $\beta$  decayers in geological minerals and meteorites) and laboratory measurements (comparisons of different transitions in atomic clocks). The astronomical methods are based mainly in the analysis of spectra from high redshift quasar absorption systems. Bounds on the variation in  $m_e$  in the early universe can be obtained using data from the Cosmic Microwave Background (CMB) radiation and from the abundances of light elements. These bounds are not as stringent as the mentioned above but they are important because they

refer to a different cosmological epoch.

In this paper, we perform a careful study of the time variation of  $m_e$  in the early universe. First, we use all available abundances of D,  $^4\text{He}$  and  $^7\text{Li}$ , the latest data from the CMB and the 2dFGRS power spectrum to put bounds on the variation in  $m_e$  without assuming any theoretical model. Afterward, we use these bounds and others from astronomical and geophysical data, to test Barrow-Magueijo theory.

In section 2, we use the abundances of the light elements to put bounds on  $\frac{\Delta m_e}{(m_e)_0}$ , where  $(m_e)_0$  is the present value of  $m_e$ , allowing the baryon to photon density  $\eta_B$  to vary. In section 3, we use the three year WMAP data, other CMB experiments and the power spectrum of the final 2dFGRS to put bounds on the variation in  $m_e$  during recombination, allowing also other cosmological parameters to vary. In sections 4, 5 and 6 we describe the astronomical and local data from the late universe. In section 7, we describe the Barrow-Magueijo model, and obtain solutions for the scalar field that drives the variation in  $m_e$ , for the early and late universe. In section 8 we show our results. Finally, in section 9 we discuss the results and summarize our conclusions.

## 2. Bounds from BBN

Big Bang Nucleosynthesis (BBN) is one of the most important tools to study the early universe. The baryon to photon ratio  $\eta_B$  or equivalently the baryon asymmetry  $\eta_B \equiv \frac{n_B - n_{\bar{B}}}{n_\gamma}$  can be determined by comparison between theoretical calculations and observations of the abundances of light elements. An independent method for determining  $\eta_B$  is provided by data from the Cosmic Microwave Background (CMB) (Spergel et al. 2003, 2007; Sanchez et al. 2006). Considering the baryon density from WMAP results, the predicted abundances are highly consistent with the observed D but not with all  $^4\text{He}$  and  $^7\text{Li}$ . Such discrepancy is usually ascribed to non reported systematic errors in the observations of  $^4\text{He}$  and  $^7\text{Li}$ . However, if the systematic errors of  $^4\text{He}$  and  $^7\text{Li}$  are correctly estimated, we may have insight into new physics beyond the minimal BBN model.

In the currently most popular particle physics models, the lepton and baryon numbers are comparable. In this case, any asymmetry between neutrinos and antineutrinos will not have a noticeable effect on the predictions of BBN. However, observational data do not imply that the lepton asymmetry should be connected to the ‘tiny’ baryon asymmetry  $\eta_B$ . Moreover, a small asymmetry between electron type neutrinos and antineutrinos can have a significant impact on BBN since the  $\nu_e$  affect the inter-conversion of neutrons to protons changing the equilibrium neutron-to-proton ratio from  $(n/p)_{eq}^0 = e^{-\frac{\Delta m}{T}}$  to  $(n/p)_{eq} =$

$(n/p)_{eq}^0 e^{-\xi_e}$  where  $\xi_e$  is the ratio of the neutrino chemical potential to the temperature. Consequently, the  ${}^4\text{He}$  abundance changes. In contrast, the D abundance is insensitive to  $\xi_e \neq 0$ . Consistent with the BBN and CMB data, values of  $\xi_e$  in the range  $-0.1 < \xi_e < 0.3$  are permitted (Barger et al. 2003; Steigman 2005, 2006). In this work, however, we assume  $\xi_e \simeq 0$  and attribute the discrepancies described above to time-variation of  $m_e$  or  $< v >$ .

We considered available observational data on D,  ${}^4\text{He}$  and  ${}^7\text{Li}$ . For D, we used the values reported by Pettini and Bowen (2001); O’Meara et al. (2001); Kirkman et al. (2003); Burles and Tytler (1998a,b); Crighton et al. (2004); O’Meara et al. (2006); Oliveira et al. (2006). For  ${}^4\text{He}$ , the available observations are reported by Peimbert et al. (2007); Izotov et al. (2007). For  ${}^7\text{Li}$ , we considered the results from Ryan et al. (2000); Bonifacio et al. (1997); Bonifacio and Molaro (1997); Bonifacio et al. (2002); Asplund et al. (2006); Boesgaard et al. (2005); Bonifacio et al. (2007). For the discussion about the consistency data check, we refer the reader to an earlier work (Mosquera et al. 2007).

We modified numerical code of Kawano (Kawano 1988; Kawano 1992) in order to allow  $m_e$  to vary. The code was also updated with the reaction rates reported by Bergström et al. (1999). The main effects of the possible variation in  $m_e$  in the physics of the first three minutes of the universe are changes in the weak rates, in the sum of electron and positron energy densities, in the sum of electron and positron pressures, and in the difference of electron and positron number densities (see appendix A for details). If  $m_e$  takes a lower value than the present one, the primordial abundances are higher than the standards. The change is more important for  ${}^4\text{He}$  and  ${}^7\text{Li}$  abundances, where a variation of 10% in  $m_e$  leads to a change of 7.4% and 8.5% in the abundances, while the effect on the D abundance is tiny (1.5%).

We computed the light nuclei abundances for different values of  $\eta_B$  and  $\frac{\Delta m_e}{(m_e)_0}$  and performed the statistical analysis to obtain the best fit values for these parameters. There is no good fit for the whole data set even for  $\frac{\Delta m_e}{(m_e)_0} \neq 0$ . However, reasonable fits can be found excluding one group of data at each time (see table 1). Figures 1 and 2 show the confidence contours and 1 dimensional Likelihoods for different groups of data. We obtained that for D +  ${}^4\text{He}$  the value of  $\eta_B$  is coincident with WMAP estimation and there is no variation in  $m_e$  within  $3\sigma$ . Moreover, the other groups of data prefer values far from WMAP estimation, and for D +  ${}^7\text{Li}$ , the result is consistent with variation in  $m_e$  within  $6\sigma$ .

As pointed out in the introduction, in the standard model, D,  ${}^4\text{He}$  and  ${}^7\text{Li}$  abundances considered separately predict very different values for the baryon density. Therefore, when the three abundances are fitted together, an intermediate value of  $\eta_B$  is obtained, but the value of  $\chi^2$  is too high. Only when two abundances are considered, we obtain a reasonable fit. Furthermore, a high variation in  $m_e$  which affects mostly the  ${}^7\text{Li}$  abundance is needed

Table 1: Best fit parameter values and  $1\sigma$  errors for the BBN constraints on  $\frac{\Delta m_e}{(m_e)_0}$  and  $\eta_B$  (in units of  $10^{-10}$ ).

	$\eta_B \pm \sigma$	$\frac{\Delta m_e}{(m_e)_0} \pm \sigma$	$\frac{\chi^2_{min}}{N-2}$
D + $^4\text{He}$ + $^7\text{Li}$	$4.237^{+0.047}_{-0.097}$	$-0.036^{+0.010}_{-0.007}$	9.33
$^4\text{He}$ + $^7\text{Li}$	$3.648^{+0.128}_{-0.124}$	$-0.055^{+0.010}_{-0.008}$	1.00
D + $^7\text{Li}$	$5.399^{+0.287}_{-0.213}$	$0.653^{+0.051}_{-0.045}$	1.01
D + $^4\text{He}$	$6.339^{+0.376}_{-0.355}$	$-0.022 \pm 0.009$	1.01

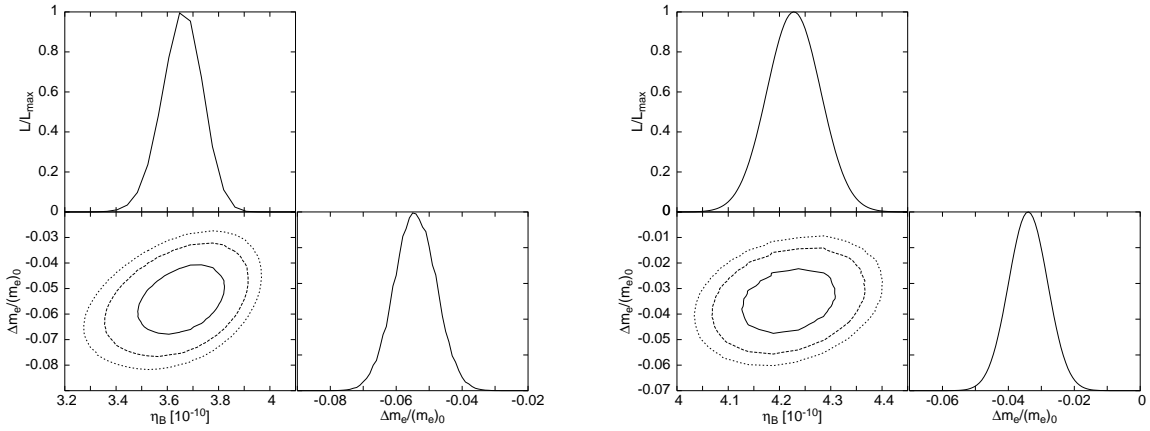


Fig. 1.—  $1\sigma$ ,  $2\sigma$  and  $3\sigma$  likelihood contours for  $\frac{\Delta m_e}{(m_e)_0}$  vs  $\eta_B$  and 1 dimensional Likelihood using  $^4\text{He} + ^7\text{Li}$  data (left) and all data (right)

to fit D and  $^7\text{Li}$  together. On the other hand, D and  $^4\text{He}$  are marginally consistent with WMAP estimation and therefore no variation in  $m_e$  is needed to fit both data at the same time. Finally, in order to fit the abundances of  $^4\text{He}$  and  $^7\text{Li}$ , a variation in  $m_e$  is needed since both quantities are affected when  $m_e$  is allowed to vary.

As mentioned in the introduction, in this paper we limit ourselves to the context of the Barrow-Magueijo model of a varying  $m_e$ . However, in more general classes of theories (Kaluza-Klein, Strings, GUTs, etc), the underlying fundamental constant is the Higgs vacuum expectation value. The dependence of the primordial abundances on the Higgs vacuum expectation value has been analyzed by Yoo and Scherrer (2003). Semi-analytical analysis have been performed by some of us in earlier works (Chamoun et al. 2007). Besides changes in  $m_e$ , a possible variation in  $\langle v \rangle$  modifies the values of the following quantities: the Fermi constant  $G_F$ , the neutron-proton mass difference  $\Delta m_{np}$ , and the deuterium binding energy  $\epsilon_D$ . The dependence of these quantities with  $\langle v \rangle$  have been described in an earlier work

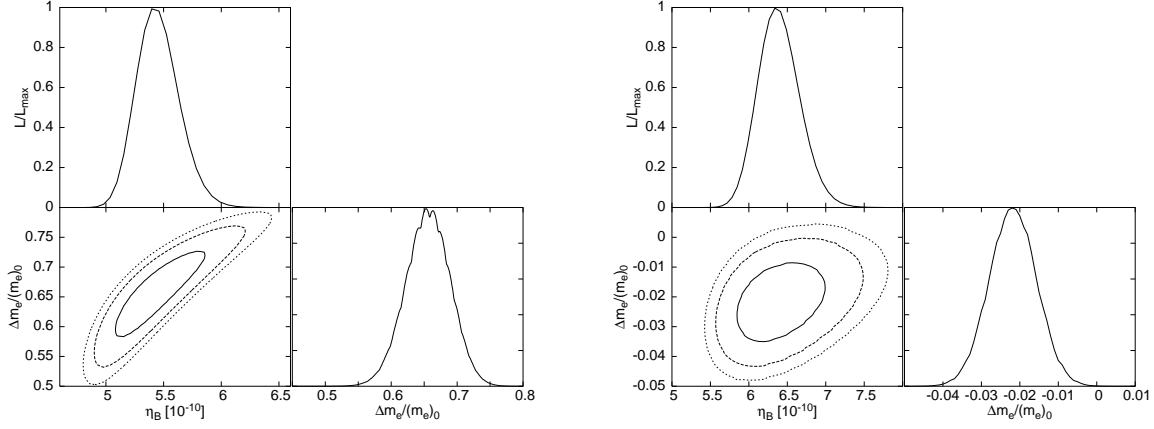


Fig. 2.— 1 $\sigma$ , 2 $\sigma$  and 3 $\sigma$  likelihood contours for  $\frac{\Delta m_e}{(m_e)_0}$  vs  $\eta_B$  and 1 dimensional Likelihood using D + <sup>7</sup>Li data (left) and D + <sup>4</sup>He (right)

(Chamoun et al. 2007) (see appendix A for details). We modified numerical code of Kawano in order to allow  $\langle v \rangle$  to vary. The abundances of the primordial elements are much higher than the standard value if the Higgs vacuum expectation value during BBN is larger than the current value. A variation of 10% in  $\langle v \rangle$  leads to a change of 45%, 25% and 29% in the <sup>4</sup>He, <sup>7</sup>Li and D abundances respectively. Since D is a residual of <sup>4</sup>He production, a great change in <sup>4</sup>He also leads to an important change in D. The changes in the abundances are greater than in the case where only  $m_e$  is allowed to vary.

In the case of  $\langle v \rangle$ , we performed the same analysis for the same groups of data we considered for  $m_e$ . As in the case of  $m_e$ , variation there is no good fit for the whole set of data. However, reasonable fits can be found excluding one group of data at each time (see table 2). Figure 3 shows the confidence contours and 1 dimensional Likelihoods for different groups of data.

Table 2: Best fit parameter values and 1 $\sigma$  errors for the BBN constraints on  $\frac{\Delta \langle v \rangle}{\langle v \rangle_0}$  and  $\eta_B$  (in units of  $10^{-10}$ ).

	$\eta_B \pm \sigma$	$\frac{\Delta \langle v \rangle}{\langle v \rangle_0} \pm \sigma$	$\frac{\chi^2_{min}}{N-2}$
D + <sup>4</sup> He + <sup>7</sup> Li	$4.275 \pm 0.097$	$0.006 \pm 0.002$	9.27
<sup>4</sup> He + <sup>7</sup> Li	$3.723^{+0.132}_{-0.124}$	$0.008 \pm 0.001$	1.00
D + <sup>7</sup> Li	$5.139^{+0.242}_{-0.231}$	$-0.138^{+0.015}_{-0.009}$	1.01
D + <sup>4</sup> He	$6.324^{+0.374}_{-0.285}$	$0.004 \pm 0.002$	1.04

We obtain that for  $D + {}^4\text{He}$  the value of  $\eta_B$  is consistent with WMAP estimation and there is no variation in  $\langle v \rangle$  within  $3\sigma$ . Moreover, the other groups of data prefer values not consistent with WMAP results. For  $D + {}^7\text{Li}$ , the result is consistent with variation in  $\langle v \rangle$  within  $6\sigma$ . The results are similar to those obtained in the case where  $m_e$  is the varying constant: i) no reasonable fit for the three abundances; ii) D and  ${}^4\text{He}$  can be well fitted with null  $\langle v \rangle$  variation; iii) D and  ${}^7\text{Li}$  need a huge variation in order to obtain a reasonable fit. However, the bounds on variation in  $\langle v \rangle$  are more stringent than the bounds obtained when only  $m_e$  was allowed to vary (see table 1). This could be explained since variations in  $\langle v \rangle$  lead to greater changes in the theoretical abundances than variation in  $m_e$ .

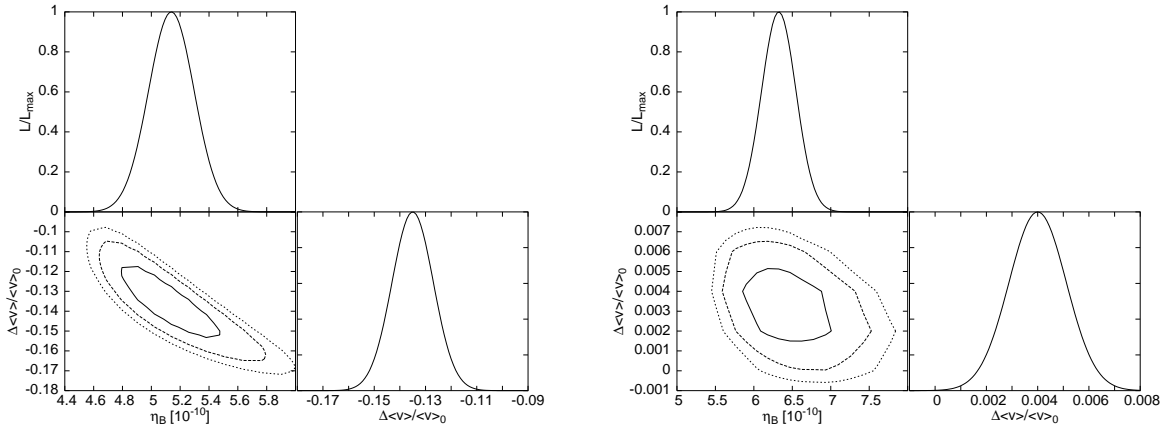


Fig. 3.—  $1\sigma$ ,  $2\sigma$  and  $3\sigma$  likelihood contours for  $\frac{\Delta\langle v \rangle}{\langle v \rangle_0}$  vs  $\eta_B$  and 1 dimensional Likelihood using  $D + {}^7\text{Li}$  (left) and  $D + {}^4\text{He}$  (right)

### 3. Bounds from CMB

Cosmic Microwave Background (CMB) radiation provides valuable information about the physical conditions of the universe just before decoupling of matter and radiation, and thanks to its dependence upon cosmological parameters, it allows their estimation. Any change in the value of  $m_e$  affects the physics during recombination, mainly the redshift of this epoch, due to a shift in the energy levels and in particular, the binding energy of hydrogen. The Thompson scattering cross section, which is proportional to  $m_e^{-2}$ , is also changed for all particles. Therefore, the CMB power spectrum is modified by a change in the relative amplitudes of the Doppler peaks, and shifts in their positions (see appendix B for details). Changes in the cosmological parameters produce similar effects. In the recombination scenario, the only effect of varying  $\langle v \rangle$  is a change in the value of  $m_e$ .

Previous analysis of the CMB data including a possible variation in  $m_e$  have been performed by Yoo and Scherrer (2003); Kujat and Scherrer (2000). In this paper, we use the WMAP 3-year temperature and temperature-polarization power spectrum (Spergel et al. 2007), and other CMB experiments such as CBI (Readhead et al. 2004), ACBAR (Kuo et al. 2004), and BOOMERANG (Piacentini et al. 2006; Jones et al. 2006), and the power spectrum of the 2dFGRS (Cole et al. 2005). We consider a spatially-flat cosmological model with adiabatic density fluctuations. The parameters of our model are:

$$P = (\Omega_B h^2, \Omega_{CDM} h^2, \Theta, \tau_{re}, \frac{\Delta m_e}{(m_e)_0}, n_s, A_s) \quad (1)$$

where  $\Omega_{CDM} h^2$  is the dark matter density in units of the critical density,  $\Theta$  gives the ratio of the comoving sound horizon at decoupling to the angular diameter distance to the surface of last scattering,  $\tau_{re}$  is the reionization optical depth,  $n_s$  the scalar spectral index and  $A_s$  is the amplitude of the density fluctuations.

We use a Markov Chain Monte Carlo method to explore the parameter space because the exploration of a multidimensional parameter space with a grid of points is computationally prohibitive. We use the public available CosmoMC code of Lewis and Bridle (2002) which uses CAMB (Lewis et al. 2000) and RECFAST (Seager et al. 1999) to compute the CMB power spectra, and we have modified them in order to include the possible variation in  $m_e$  at recombination. We ran eight different chains. We used the convergence criterion of Raftery and Lewis (1992) to stop the chains when  $R - 1 < 0.0044$  (more stringent than the usually adopted value). Results are shown in table 3 and figure 4. Figure 4 shows a strong degeneracy between  $m_e$  and  $\Theta$ , which is directly related to  $H_0$ , and also between  $m_e$  and  $\Omega_B h^2$ , and  $m_e$  and  $\Omega_{CDM} h^2$ .

We have also performed the analysis considering only CMB data. The strong degeneracy between  $m_e$  and  $H_0$  made the chains cover all the wide  $H_0$  prior, making it impossible to

Table 3: Mean values and errors for the main and derived parameters including  $m_e$  variation ( $H_0$  is in units of  $\text{km s}^{-1} \text{Mpc}^{-1}$ ).

Parameter	Mean value and $1\sigma$ error	Parameter	Mean value and $1\sigma$ error
$\Omega_B h^2$	$0.0217 \pm 0.0010$	$\Omega_{CDM} h^2$	$0.1006^{+0.0085}_{-0.0086}$
$\Theta$	$1.020 \pm 0.025$	$\tau_{re}$	$0.091^{+0.013}_{-0.014}$
$\frac{\Delta m_e}{(m_e)_0}$	$-0.029 \pm 0.034$	$n_s$	$0.960 \pm 0.015$
$A_s$	$3.020 \pm 0.064$	$H_0$	$68.1^{+5.9}_{-6.0}$



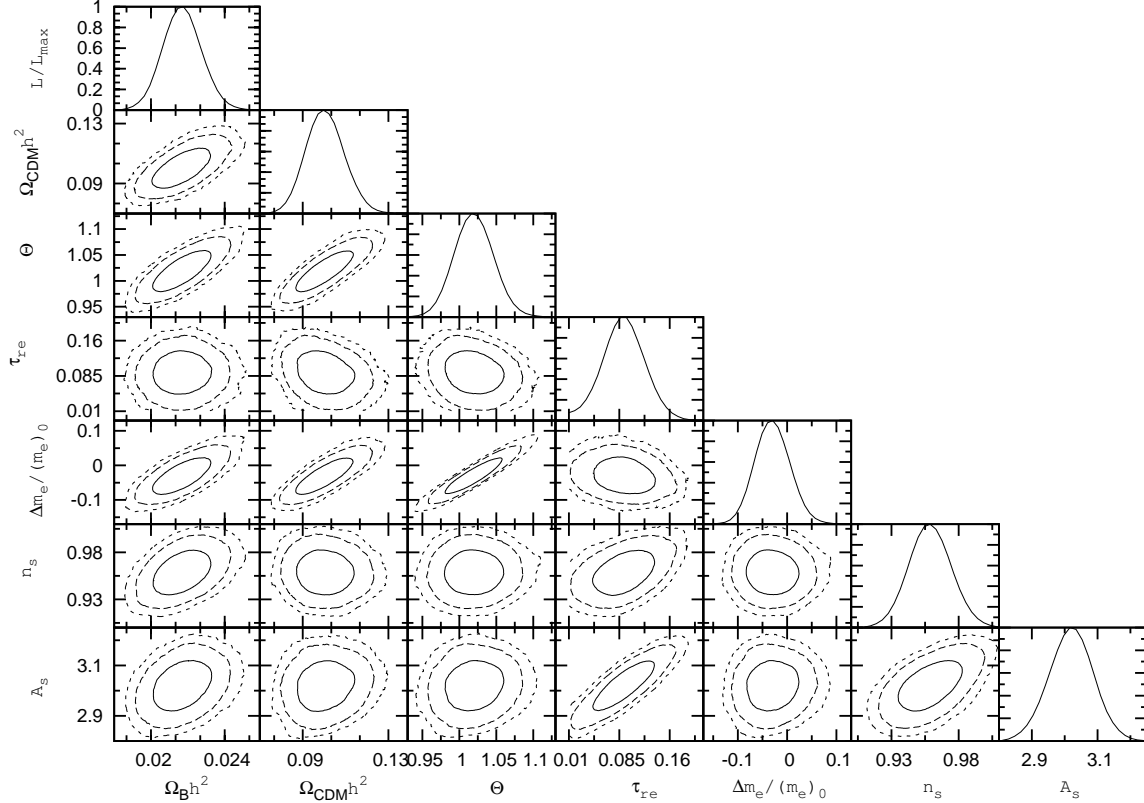


Fig. 4.— Marginalized posterior distributions obtained with CMB data, including the WMAP 3-year data release plus 2dFGRS power spectrum. The diagonal shows the posterior distributions for individual parameters, the other panels shows the 2D contours for pairs of parameters, marginalizing over the others.

find reliable mean values and errors. Hence, we added a gaussian prior to  $H_0$ , which was obtained from the Hubble Space Telescope Key Project (Freedman et al. 2001), and chose the values of the mean and errors as those inferred from the closest objects in that paper, so we could neglect any possible difference between the value of  $m_e$  at that redshift and the present value. In this way, we post-processed the chains and found limits that are consistent with those of the first analysis, revealing the robustness of these bounds. However, the most stringent constraints were obtained in the first analysis (see figure 5).

Finally, we comment that *Planck* will be the first mission to map the entire CMB sky with mJy sensitivity and resolution better than 10' (The Planck Collaboration 2006). Such

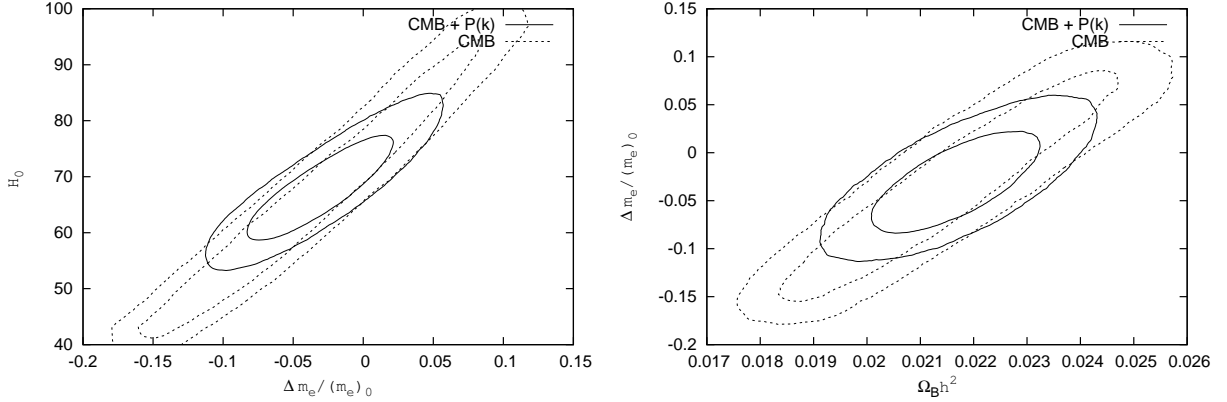


Fig. 5.—  $1\sigma$ , and  $2\sigma$  confidence levels contours obtained with CMB data with and without data of the 2dFGRS power spectrum.

resolution will allow to see into the damping tail of the anisotropy spectrum, around the third and fourth peaks, with a precision almost only limited by cosmic variance (White 2006). This will enable a very precise estimation of the baryon-to-photon ratio from the relative height of the peaks in the spectrum.

#### 4. Bounds from Quasar Absorption Systems

Quasar absorption systems present ideal laboratories to search for any temporal variation of the fundamental constants over cosmological time scales. In particular, a method for constraining the variation in  $\mu = \frac{m_p}{m_e}$  was developed by Varshalovich and Levshakov (1993). It is based on the fact that wavelengths of electron-vibro-rotational lines depend on the reduced mass of the molecules, with different dependence for different transitions. In such way, it is possible to distinguish the cosmological redshift of a line from the shift caused by a variation in  $\mu$ . The rest-frame laboratory wavelength,  $\lambda_i^0$ , can be related to those in the quasar absorption system,  $\lambda_i$ , as  $\frac{\lambda_i}{\lambda_i^0} = (1 + z_{abs})(1 + K_i \frac{\Delta\mu}{\mu})$ , where  $z_{abs}$  is the absorption redshift and  $K_i$  is the coefficient which determines the sensitivity of the wavelength  $\lambda_i$ . Using observations from  $H_2$  absorption systems at high redshift and laboratory measurements, several authors obtained constraints on  $\mu$  (Potekhin et al. 1998; Levshakov et al. 2002; Ivanchik et al. 2003, 2005). The most up-to-date available measurements for each redshift are listed in table 4 and will be considered to test Barrow-Magueijo model.

Another method for constraining variation in fundamental constants is based on the comparison between the hyperfine 21 cm absorption transition of neutral hydrogen ( $\nu_a$ ) and

Table 4: The table shows the absorption redshift, the value of  $\frac{\Delta m_e}{(m_e)_0}$  with its corresponding error (in units of  $10^{-5}$ ), and the reference obtained comparing molecular and laboratory wavelengths.

Redshift	$\frac{\Delta m_e}{(m_e)_0} \pm \sigma$	Reference
2.8	$-6.25 \pm 13.70$	Potekhin et al. (1998)
3.02	$-1.40 \pm 0.83$	Ivanchik et al. (2005)
2.6	$-2.11 \pm 1.39$	Ivanchik et al. (2005)

an optical resonance transition ( $\nu_b$ ). The ratio  $\frac{\nu_a}{\nu_b}$  is proportional to  $x = \alpha^2 g_p \frac{m_e}{m_p}$  where  $g_p$  is the proton  $g$  factor (Tzanavaris et al. 2007). Thus, a change of this quantity will result in a difference in the redshift measured from 21 cm and optical absorption lines as  $\frac{\Delta x}{x} = \frac{z_{opt} - z_{21}}{(1+z_{21})}$ . Since we are working in the context of the Barrow and Magueijo (2005) model, the only fundamental constant which is allowed to vary is  $m_e$ . Table 5 shows the bounds obtained by Tzanavaris et al. (2007) combining the measurements of optical and radio redshift. This method has the inconvenience that it is difficult to determine if both radio and optical lines were originated at the same absorption system. Thus, a difference in the velocity of the absorption clouds could hide a variation in  $x$ .

Table 5: The table shows the absorption redshift, the value of  $\frac{\Delta m_e}{(m_e)_0}$  with its corresponding error (in units of  $10^{-5}$ ), obtained comparing radio and molecular redshifts (Tzanavaris et al. 2007).

Redshift	$\frac{\Delta m_e}{(m_e)_0} \pm \sigma$	Redshift	$\frac{\Delta m_e}{(m_e)_0} \pm \sigma$
0.24	$1.21 \pm 2.10$	1.78	$-2.59 \pm 0.90$
0.31	$-0.61 \pm 4.27$	1.94	$3.30 \pm 0.44$
0.40	$3.22 \pm 3.15$	2.04	$5.20 \pm 2.76$
0.52	$-2.95 \pm 1.05$	2.35	$-2.54 \pm 1.82$
0.52	$0.26 \pm 3.67$		

## 5. Bounds from Geophysical Data

The half-life of long-lived  $\beta$  decayers has been determined either in laboratory measurements or by comparison with the age of meteorites, as found from  $\alpha$  decay radioactivity analysis. The most stringent bound on the variation in the half life,  $\lambda$ , proceeds from the

comparison of  $^{187}\text{Re}$  decay in the Solar System formation and the present (Olive et al. 2004):  $\frac{\Delta\lambda}{\lambda} = (-0.016 \pm 0.016)$ . Sisterna and Vucetich (1990) derived a relation between the shift in the half-life of long lived  $\beta$  decayers and a possible variation between the values of the fundamental constants values now. In this paper, we only consider  $m_e$  variation and therefore  $\frac{\Delta\lambda}{\lambda} = a \frac{\Delta m_e}{(m_e)_0}$ , where  $a = -600$  for  $^{187}\text{Re}$ .

## 6. Bounds from Laboratory

Comparison between frequencies of different atomic transitions over time are useful tools to put stringent bounds on the variation in fundamental constants at present. In particular, the hyperfine frequency of cesium can be approximated by  $\nu_{\text{Cs}} \simeq g_{\text{Cs}} \frac{m_e}{m_p} \alpha^2 R_y F_{\text{Cs}}(\alpha)$  (where  $g_{\text{Cs}}$  is the nuclear  $g$  factor,  $R_y$  is the Rydberg constant expressed as a frequency and  $F_{\text{Cs}}(\alpha)$  is a dimensionless function of  $\alpha$  and does not depend on  $m_e$  at least at first order), while optical transition frequencies can be expressed as  $\nu_{\text{opt}} \simeq R_y F(\alpha)$ . Several authors (Bize et al. 2003; Fischer et al. 2004; Peik et al. 2004) have measured different optical transitions and compared them with the frequency of the ground state hyperfine splitting in neutral  $^{133}\text{Cs}$ . These measurements can be used to constrain the variation in  $\frac{\dot{m}_e}{(m_e)_0}$ . Constraints from different experiments are listed in table 6.

Table 6: The table shows the compared clocks, the value of  $\frac{\dot{m}_e}{(m_e)_0}$  with its corresponding error (in units of  $10^{-15}$ ), the time interval for which the variation was measured and the reference.

Frequencies	$\frac{\dot{m}_e}{(m_e)_0} \pm \sigma [10^{-15}\text{yr}^{-1}]$	$\Delta t[\text{yr}]$	Reference
Hg <sup>+</sup> and Cs	$0.2 \pm 7.0$	5	Fischer et al. (2004)
Yb <sup>+</sup> and Cs	$1.2 \pm 4.4$	2.8	Peik et al. (2004)
Hg <sup>+</sup> and Cs	$0 \pm 7$	2	Bize et al. (2003)

## 7. The Model

We now analyze the Barrow-Magueijo model for the variation in  $m_e$ . We solve the equation of the scalar field ( $\phi$ ) that drives the variation in  $m_e$  in this model. We consider that the variations in  $\phi$  are small and they do not produce significant contributions to the Friedmann equation. As we did in a previous work (Mosquera et al. 2007), we build a piecewise approximate solution by joining solutions obtained by keeping only some terms of the Friedmann equation, relevant in the following domination regimes: a) radiation and

matter, and b) matter and cosmological constant. In such way, solution a) can be applied to nucleosynthesis and recombination of primordial hydrogen whereas solution b) is appropriate for quasar absorption systems, geophysical data and atomic clocks.

Defining the variable  $\vartheta$  as  $d\vartheta = d\tau/a$ , where  $\tau = H_0 t$ , and  $t$  is the cosmic time, the expression for the scale factor in the radiation and matter regime is:

$$a_{RM}(\vartheta) = \frac{\Omega_m}{4}\vartheta^2 + \sqrt{\Omega_r}\vartheta \quad (2)$$

and the relationship between  $\tau$  and  $\vartheta$  is:

$$\tau(\vartheta) = \frac{\Omega_m}{12}\vartheta^3 + \frac{\sqrt{\Omega_r}}{2}\vartheta^2 \quad (3)$$

The solution for the scale factor in the matter and cosmological constant regime can be written as:

$$a_{MC}(\tau) = \left(\frac{\Omega_m}{\Omega_\Lambda}\right)^{1/3} \left[ \sinh \left( \frac{3}{2} \sqrt{\Omega_\Lambda} (\tau - \tau_0) + \sinh^{-1} \sqrt{\frac{\Omega_\Lambda}{\Omega_m}} \right) \right]^{2/3} \quad (4)$$

where  $\tau_0 = H_0 t_0$ , and  $t_0$  is the age of the universe. To obtain the last solution we have considered that the scale factor must be a continuous and smooth function of time.

In the Barrow-Magueijo model,  $m_e$  is controlled by a dilaton field  $\phi$  defined by  $m_e = (m_e)_0 \exp(\phi)$ , and variations in  $m_e$  occur no sooner the universe cools down below  $m_e$  threshold. The minimal dynamics for  $\phi$  is set by the kinetic Lagrangian

$$\mathcal{L}_\phi = \frac{\omega}{2} \partial_\mu \phi \partial^\mu \phi \quad (5)$$

where  $\omega$  is a coupling constant. From this Lagrangian, the equation of motion of the scalar field can be derived as:

$$(\dot{\phi} a^3) = -M \exp[\phi] \quad (6)$$

with  $M \simeq \rho_{e0} a_0^3 c^4 / \omega$ . This is a second order equation for  $\phi$ , with the boundary condition  $\phi_0 = 0$ . If the mass variations are small,  $e^\phi \simeq 1$  can be set to obtain an analytical expression for  $\phi$ .

For convenience, we define  $\beta = \frac{1}{4} \Omega_m \Omega_r^{-1/2}$ ,  $\xi = \Omega_m \Omega_r^{-3/4} \Omega_\Lambda^{-1/4}$ ,  $\gamma = \Omega_\Lambda^{1/2} \Omega_m^{-1/2}$  and

$$\begin{aligned} f(\xi) &= \frac{2 + (\xi - 2)\sqrt{1 + \xi}}{\xi^2} \\ C &= \sinh^{-1} \xi^{-1/2} - f(\xi) \end{aligned} \quad (7)$$

Provided  $\phi = \ln(m_e/(m_e)_0) \simeq \Delta m_e/(m_e)_0$ , the expressions for the variation in  $m_e$  in the two regimes are:

- for  $\tau < \tau_1$  (where  $\tau_1$  is defined by  $a_{RM}(\tau_1) = a_{MC}(\tau_1) = \left(\frac{\Omega_r}{\Omega_\Lambda}\right)^{1/4}$ , see Mosquera et al. (2007)):

$$\begin{aligned} \frac{\Delta m_e}{(m_e)_0}(\vartheta) = & \frac{2}{3} \frac{M}{H_0^2} \frac{1}{\Omega_m} \left[ -2 \ln \left( \frac{2(\beta\vartheta + 1)}{1 + \sqrt{1 + \xi}} \right) + \frac{1}{\beta\vartheta + 1} - \frac{2}{1 + \sqrt{1 + \xi}} + \frac{2}{3} f(\xi) \sqrt{1 + \xi} \right. \\ & \left. + \frac{1}{4} \ln \left( \frac{\Omega_\Lambda}{\Omega_r} \right) - \frac{2}{3} (\sinh^{-1} \gamma - C) \frac{\sqrt{1 + \gamma^2}}{\gamma} \right] \\ & + \frac{A}{H_0} \frac{\Omega_m}{\Omega_r^{3/2}} \left[ \frac{1}{2} \ln \left( \frac{\beta\vartheta + 1}{\beta\vartheta} \right) + \frac{1}{2} \ln \left( \frac{\sqrt{1 + \xi} - 1}{\sqrt{1 + \xi} + 1} \right) - \frac{1}{4\beta\vartheta} - \frac{1}{4(\beta\vartheta + 1)} \right. \\ & \left. + \frac{(\xi - \frac{2}{3}) \sqrt{1 + \xi} + \frac{2}{3} \frac{\sqrt{1 + \gamma^2}}{\gamma}}{\xi^2} \right] \end{aligned} \quad (8)$$

The relationship between  $\tau$  and  $\vartheta$  is given by Eq.(3).

- for  $\tau > \tau_1$

$$\begin{aligned} \frac{\Delta m_e}{(m_e)_0}(\tau) = & \phi_0 + \frac{M}{H_0^2} \frac{2}{3\Omega_m} \left[ \sqrt{\Omega_\Lambda} \tau \coth \left( C + \frac{3}{2} \sqrt{\Omega_\Lambda} \tau \right) - \frac{2}{3} \ln \left[ \sinh \left( C + \frac{3}{2} \sqrt{\Omega_\Lambda} \tau \right) \right] \right. \\ & \left. + \frac{2}{3} \left( \ln \gamma - \frac{\sqrt{1 + \gamma^2}}{\gamma} \left[ C + \ln \left( \gamma + \sqrt{1 + \gamma^2} \right) \right] \right) \right] + \\ & + \frac{A}{H_0} \frac{2\sqrt{\Omega_\Lambda}}{3\Omega_m} \left[ -\coth \left( C + \frac{3}{2} \sqrt{\Omega_\Lambda} \tau \right) + \frac{\sqrt{1 + \gamma^2}}{\gamma} \right] \end{aligned} \quad (9)$$

where  $A$  is an integration constant.

## 8. Results

The model described in section 7 predicts the variation in  $m_e$  as a function of time, and has two independent dimensionless parameters  $M/H_0^2$  and  $A/H_0$ . We do not fix  $A/H_0$  to zero as previous works did (Barrow and Magueijo 2005). To constrain these parameters, we use the data described in the previous sections. We perform a  $\chi^2$  test to obtain the best fit parameters of the model. In order to obtain the parameters consistently with our assumption that the energy density of the field  $\phi$  ( $\epsilon_\phi = \frac{1}{c^2} \frac{\omega}{2} \dot{\phi}^2$ ) can be neglected in the Friedmann equation, we add to the  $\chi^2$  expression, a term that controls that the contribution of  $\phi$  to the Friedmann equation will be less important than the radiation term, right after  $m_e$  threshold is crossed. The result of the statistical analysis shows that there is no good fit

for the whole data set. We repeat the analysis excluding one group of data at each time. We found that reasonable fits can be obtained excluding the quasar at  $z = 1.94$  of table 5 and that the data from nucleosynthesis is crucial to determine the value of  $A/H_0$ . Besides, the group of data from table 4 is important to determine the value of  $M/H_0^2$ . The results are shown in table 7.

Table 7: The table shows the best fit parameters of the model (excluding entry 7 of table 5). The value for the  $M/H_0^2$  parameter is in units of  $10^{-6}$ , and the value for the  $A/H_0$  parameter is in units of  $10^{-13}$ .

$M/H_0^2$	$A/H_0$	$G\omega/c^4$	$\frac{\chi_{min}^2}{N-2}$
$-7.30^{+2.10}_{-2.02}$	$3.60^{+1.44}_{-1.50}$	$-0.336^{+0.097}_{-0.093}$	1.14

Since the Barrow-Magueijo model is written in terms of the coupling constant  $\omega$ , we derive its best value from the previous constraints. Since  $M \simeq \rho_{e0} a_0^3 c^4 / \omega$  we obtain the following relationship:

$$\frac{G\omega}{c^4} = \frac{3}{8\pi} \left(1 - \frac{f_{\text{He}}}{2}\right) \Omega_b \frac{m_e}{m_p} \left(\frac{M}{H_0^2}\right)^{-1} \quad (10)$$

where  $f_{\text{He}}$  is the fraction of the total number of baryons in the form of He, and can be written as a function of the total observed mass abundance of He ( $M_{\text{He}}/M_{\text{H}}$ ). According to the values of  $M/H_0^2$  from table 7 and using  $f_{\text{He}} = 0.19$  (taking  $M_{\text{He}}/M_{\text{H}} = 0.24$ ), we obtain the bounds on the dimensionless quantity  $\frac{G\omega}{c^4}$  presented in table 7.

## 9. Summary and Conclusions

In this paper, we have put limits on the time variation in the electron mass at primordial nucleosynthesis time using observational primordial abundances of D,  $^4\text{He}$  and  $^7\text{Li}$ , and we have analyzed in detail the consequences of considering different groups of data. We have also considered the variations in  $\langle v \rangle$  during BBN and analyzed the differences with the variations in  $m_e$  during the same epoch. Additionally, we have used the three year data from the Cosmic Microwave Background and the final 2dFGRS power spectrum to obtain bounds on the variation in  $m_e$  at recombination, and an estimation of the cosmological parameters. Together with other bounds on the variation in the late universe, that come from quasar absorption systems, half-life of long-lived  $\beta$  decayers, and atomic clocks, we put constraints on the Barrow-Magueijo model for the variation in  $m_e$ . We have improved the solutions by

taking into account the detailed evolution of the scale factor and the complete solution for the scalar field that drives the variation in  $m_e$ .

In the original paper (Barrow and Magueijo 2005) some approximations in the evolution of the scale factor are assumed with the consequent simplification in the solution for the scalar field. Another improvement of our derivation is that we have not neglected the first integration constant, which is the most contributing part in the early universe. In fact, integrating Eq.(6) once, we can write:

$$\dot{\phi}a^3 = -M \left( t - \frac{A}{M} \right) = -M (t + 8.47 \times 10^2 \text{yr}) \quad (11)$$

where we have used the best fit values for the parameters  $M/H_0^2$  and  $A/H_0$ , and  $h = 0.73$ . Note that the second term in the right hand side of Eq.(11) is dominant in the early universe, in particular, during nucleosynthesis.

Barrow and Magueijo (2005) presented a bound of  $G|\omega| > 0.2$  (with  $c = 1$ ). They obtained such constraint using bounds from quasars at  $z \sim 1$ , whereas we use all the available bounds on the variation in  $m_e$  at different cosmological times. In appendix C we briefly discuss the difference in both analysis. From data supporting the weak equivalence principle, they obtain  $G|\omega| > 10^3$  while we obtain  $-0.615 < G\omega/c^4 < -0.045$  ( $3\sigma$  interval) using data from different cosmological time scales. More research both on time variation data and on the bound from WEP is needed to understand this discrepancy.

Finally, we remark that, at  $2\sigma$ , the value of  $\omega$  is negative. This should not be surprising. Indeed, negative kinetic terms in the Lagrangian have already been considered in k-essence models with a phantom energy component (Caldwell 2002).

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## A. Physics at BBN

In this appendix we discuss the dependences of the physical quantities involved in the calculation of the abundances of the light elements with  $m_e$  and variation in the Higgs vacuum expectation value. We also discuss how this quantities are changed within the Kawano Code.

### A.1. Variation in the electron mass

A change in the value of  $m_e$  at the time of primordial nucleosynthesis with respect to its present value affects derived physical quantities such as the sum of the electron and positron energy densities, the sum of the electron and positron pressures and the difference of the electron and positron number densities. In Kawano's code, these quantities are calculated as follows:

$$\rho_{e-} + \rho_{e+} = \frac{2}{\pi^2} \frac{(m_e c^2)^4}{(\hbar c)^3} \sum_n (-1)^{n+1} \text{ch}(n\phi_e) M(nz) \quad (\text{A1})$$

$$\frac{p_{e-} + p_{e+}}{c^2} = \frac{2}{\pi^2} \frac{(m_e c^2)^4}{(\hbar c)^3} \sum_n \frac{(-1)^{n+1}}{nz} \text{ch}(n\phi_e) N(nz) \quad (\text{A2})$$

$$\frac{\pi^2}{2} \left[ \frac{\hbar c^3}{m_e c^2} \right]^3 z^3 (n_{e-} - n_{e+}) = z^3 \sum_n (-1)^{n+1} \text{sh}(n\phi_e) L(nz) \quad (\text{A3})$$

where  $z = \frac{m_e c^2}{kT_\gamma}$ ,  $\phi_e$  is the electron chemical potential and  $L(z)$ ,  $M(z)$ , and  $N(z)$  are related to the modified Bessel function (Kawano 1988; Kawano 1992). In order to include the variation in  $m_e$  we replace, in all the equations,  $m_e$  by  $(m_e)_0 \left(1 + \frac{\Delta m_e}{(m_e)_0}\right)$ . The change in these quantities, due to a change in  $m_e$ , affects their derivatives and the expansion rate through the Friedmann equation. The  $n \leftrightarrow p$  reaction rates and the other weak decay rates are changed if  $m_e$  varies with time. The total  $n \rightarrow p$  reaction rate is calculated by:

$$\lambda = K \int_{m_e}^{\infty} \frac{dE_e E_e p_e (E_e + \Delta m_{np})^2}{(1 + e^{E_e/T_\gamma}) (1 + e^{-(E_e + \Delta m_{np})/T_\nu - \xi_l})} + K \int_{m_e}^{\infty} \frac{dE_e E_e p_e (E_e - \Delta m_{np})^2}{(1 + e^{-E_e/T_\gamma}) (1 + e^{(E_e - \Delta m_{np})/T_\nu - \xi_l})} \quad (\text{A4})$$

where  $E_e$  and  $p_e$  are the electron energy and momentum respectively,  $\Delta m_{np}$  is the neutron-proton mass differences,  $K$  is a normalization constant proportional to  $G_F^2$ , and  $E_e = (p_e^2 + m_e^2)^{1/2}$ .

It is worth to mention that the most important changes in the primordial abundances (due to a change in  $m_e$ ) arrives from the change in the weak rates rather than the change in the expansion rate (Yoo and Scherrer 2003).

## A.2. Variation in the Higgs vacuum expectation value

If the the value of  $\langle v \rangle$  during BBN is different than its present value, the electron mass, the Fermi constant, the neutron-proton mass difference and the deuterium binding energy take different values than the current ones. The electron mass is proportional to the Higgs vacuum expectation value. The Fermi constant is proportional to  $\langle v \rangle^{-2}$  (Dixit and Sher 1988). This dependence affect the  $n \leftrightarrow p$  reaction rates since  $K \sim G_F^2$ . The neutron-proton mass difference changes by (Christiansen et al. 1991)

$$\frac{\delta \Delta m_{np}}{\Delta m_{np}} = 1.587 \frac{\Delta \langle v \rangle}{\langle v \rangle_0} \quad (\text{A5})$$

affecting  $n \leftrightarrow p$  reaction rates (see Eq.(A4)) and the initial neutrons and protons abundances:

$$Y_n = \frac{1}{1 + e^{\Delta m_{np}/T_9 + \xi}} \quad Y_p = \frac{1}{1 + e^{-\Delta m_{np}/T_9 - \xi}} \quad (\text{A6})$$

where  $T_9$  is the temperature in units of  $10^9$  K. In order to include these effects we replace  $\Delta m_{np}$  by  $\Delta m_{np} \left(1 + \frac{\delta \Delta m_{np}}{\Delta m_{np}}\right)$ . The deuterium binding energy must also be corrected by  $\frac{\Delta \epsilon_D}{\epsilon_D} = k \frac{\Delta \langle v \rangle}{\langle v \rangle_0}$  where  $k$  is a model dependent constant. In this work we assume, following Chamoun et al. (2007),  $k = -0.045$ . This correction affects the initial value of the deuterium abundance. Once again we replace  $\epsilon_D$  by  $\epsilon_D \left(1 + \frac{\Delta \epsilon_D}{\epsilon_D}\right)$  in the code.

## B. Physics at recombination epoch

During recombination epoch, the ionization fraction,  $x_e$ , is determined by the balance between photoionization and recombination. The recombination equation is

$$-\frac{d}{dt} \left( \frac{n_e}{n} \right) = C \left( \frac{\alpha_c n_e^2}{n} - \beta_c \frac{n_{1s}}{n} e^{-(B_1 - B_2)/kT} \right) \quad (\text{B1})$$

where

$$C = \frac{(1 + K \Lambda_{2s,1s} n_{1s})}{(1 + K(\beta_c + \Lambda_{2s,1s}) n_{1s})} \quad (\text{B2})$$

is the Peebles factor, which inhibits the recombination rate due to the presence of Lyman- $\alpha$  photons. The redshift of these photons is  $K = \frac{\lambda_\alpha^3 a}{8\pi \dot{a}}$ , with  $\lambda_\alpha = \frac{8\pi \hbar c}{3B_1}$ , and  $\Lambda_{2s,1s}$  is the rate of decay of the  $2s$  excited state to the ground state via 2-photon emission, and scales as  $m_e$ . Recombination directly to the ground state is strongly inhibited, so the *case B* recombination takes place. The *case B* recombination coefficient  $\alpha_c$  is proportional to  $m_e^{-3/2}$ .

The photoionization coefficient depends on  $\alpha_c$ , but it also has an additional dependence on  $m_e$ ,

$$\beta_c = \alpha_c \left( \frac{2\pi m_e k T_m}{h^2} \right)^{3/2} e^{-B_2/kT_m} \quad (\text{B3})$$

The most important effects of a change in  $m_e$  during recombination would be due to its influence upon Thomson scattering cross section  $\sigma_T = \frac{8\pi}{3} \frac{\hbar^2 c^2}{m_e^2} \alpha^2$ , and the binding energy of hydrogen  $B_1 = \frac{1}{2} \alpha^2 m_e c^2$ .

### C. Different limits on $G\omega$

In this appendix we compare the limits obtained by Barrow and Magueijo (2005) on  $G\omega$  with our bounds. We stress that we have performed a  $\chi^2$  using all available observational and experimental data, while Barrow and Magueijo (2005) consider  $|\frac{\Delta\mu}{\mu}| < 10^{-5}$  for data at redshift of order 1. Moreover, most exact individual bounds from quasar absorption systems are not consistent with null variation at least at  $1\sigma$ .

Let us consider for example the last entry of table 4:  $-3.5 \times 10^{-5} < \frac{\Delta\mu}{\mu} < -0.72 \times 10^{-5}$ . Using the same approximation as Barrow and Magueijo (2005), we find that  $-0.28 < G\omega < -0.05$  for this measurement which is of the same order of magnitude as obtained considering all data.

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